Solutions for Exam Physics Laboratory 1: Data and error analysis
30 October 2013

Exercise 1 *(4 points total)*

a) $v = 2718 \pm 2$ mm/s $= 2.718 \pm 0.002$ m/s *(1 point)*

b) $L = 3.14000 \pm 0.00002$ km $= 3140.00 \pm 0.02$ m $= 314000 \pm 2$ cm *(1 point)*

c) $C = 4.7 \pm\ 0.5$ mF $= (4.7 \pm 0.5) \cdot 10^3\ \mu$F *(1 point)*

d) $R = 68.00 \pm 0.03$ MΩ *(1 point)*

Exercise 2 *(5 points total)*

$R = 330\ \Omega$ and $I = 0.28$ A $\Rightarrow P = I^2R = 25.872$ W (intermediate result $\rightarrow$ keep more decimals). *(1 point)*

$\Delta R/R = 5\% = 0.05, \Delta I/I = 0.01/0.28 = 0.0357 \Rightarrow$

$$\begin{aligned}\left(\frac{\Delta P}{P}\right)^2 &= \left(\frac{\Delta\{I^2\}}{\{I^2\}}\right)^2 + \left(\frac{\Delta R}{R}\right)^2 = \left(2\frac{\Delta I}{I}\right)^2 + \left(\frac{\Delta R}{R}\right)^2 \\ &= 4\left(\frac{\Delta I}{I}\right)^2 + \left(\frac{\Delta R}{R}\right)^2 = 0.00760204\end{aligned}$$

(2 points for correct formula)

$\Rightarrow \Delta P/P = 0.0872 \approx 9\% \Rightarrow \Delta P = 2.2558 \approx 2.3$ W . *(1 point)*

$\Rightarrow P \pm \Delta P = 26 \pm 3$ W. *(1 point)*

Exercise 3 *(5 points total)*

$$\begin{aligned} w_1 = \frac{1}{s_1^2} = \frac{1}{0.5^2} = 4 \quad &\text{and} \quad w_2 = \frac{1}{s_2^2} = \frac{1}{0.2^2} = 25 \quad (1\ point) \\ L = \frac{w_1L_1 + w_2L_2}{w_1 + w_2} \quad &= \quad \frac{4 \cdot 16.4 + 25 \cdot 16.1}{4 + 25} = 16.1414 \quad (1\ point) \\ \frac{1}{s_L^2} = \frac{1}{s_1^2} + \frac{1}{s_2^2} = w_1 + w_2 \quad &= \quad 4 + 25 = 29 \Rightarrow s_L = 29^{-1/2} = 0.1857 = \Delta L \quad (2\ points)\end{aligned}$$

Correct notation: $L \pm \Delta L = 16.1 \pm 0.2$ m. *(1 point)*

Exercise 4 *(10 points total)*

a) $\overline{R} = (1/6) \cdot (47.1 + 47.4 + 47.8 + 46.9 + 47.2 + 47.5) = 47.3167\ \Omega \approx 47.3\ \Omega$. *(2 points)*

b) $s^2 = \frac{1}{N-1}\sum_{i=1}^{6}(R_i - \overline{R})^2$
$= \frac{1}{5}((-0.2167)^2 + 0.0833^2 + 0.4833^2 + (-0.4167)^2 + (-0.1167)^2 + 0.1833^2)$
$= 0.1017 \Rightarrow \sigma = s = \sqrt{0.1017} = 0.3189\ \Omega \approx 0.3\ \Omega$. *(2 points)*

c) $s_m = s/\sqrt{N} = 0.3189/\sqrt{6} = 0.1302\ \Omega \approx 0.2\ \Omega$. *(2 points)*

d) N needs to be 9 times higher ($\sqrt{9} = 3$); $9 \times 6 = 54$, so 54 - 6 = 48 *extra* measurements are needed. *(2 points)*

e) s_m is the best estimate for the standard error in $\overline{R}$. The probability of finding a result within $\pm s_m$ from $\overline{R}$ is 68%. *(2 points)*

Exercise 5 *(11 points total)*

x	$y \pm \Delta y$	r
1.00	10 ± 2	-1
2.00	22 ± 2	1
3.00	32 ± 2	1
4.00	40 ± 2	-1

$$a = \frac{N\sum x_i y_i - \sum x_i \sum y_i}{N\sum x_i^2 - \left(\sum x_i\right)^2},$$

$$(\Delta a)^2 = \left(\frac{1}{\sum x_i^2 - N\overline{x}^2}\right)\frac{\sum r_i^2}{N-2},$$

$$(\Delta b)^2 = \left(\frac{1}{N} + \frac{\overline{x}^2}{\sum x_i^2 - N\overline{x}^2}\right)\frac{\sum r_i^2}{N-2}.$$

a) $N = 4, \sum x_i = 10, \sum y_i = 104, \sum x_i^2 = 30, \sum x_i y_i = 310$
$\Rightarrow a = (4 \cdot 310 - 10 \cdot 104)/(4 \cdot 30 - \{10\}^2) = 10$
and $\overline{x} = 2.5, \overline{y} = 26 \Rightarrow b = \overline{y} - a\overline{x} = 1$. *(2 points)*

b) $\sum r_i^2 = 4 \Rightarrow (\Delta a)^2 = (30 - 4 \cdot 2.5^2)^{-1}(4/\{4-2\}) = 0.4$
$\Rightarrow \Delta a = 0.63246 \approx 0.7$
and $(\Delta b)^2 = (1/4 + 2.5^2/\{30 - 4 \cdot 2.5^2\}(4/\{4-2\}) = 3$
$\Rightarrow \Delta b = 1.7321 \approx 2$. *(2 points)*

c) $\chi^2_{obs} = \sum\{r_i^2/(\Delta y_i)^2\} = 4 \cdot (1/4) = 1$. *(3 points)*

d) $\nu = N - 2 = 2$, 10% level $\Rightarrow \chi^2_{table} = 0.211$, 90% level $\Rightarrow \chi^2_{table} = 4.604$. χ^2_{obs} is in between those two limits, so the linear fit is acceptable. *(2 points)*

e) Now $\Delta y = 0.5 \Rightarrow \chi^2_{obs} = \sum\{r_i^2/(\Delta y_i)^2\} = 4/0.5^2 = 16$, which is outside the limits found in d), so in this case the linear fit is not acceptable. *(2 points)*

Exam grade = (total of points) / 4 + 1.25